

IN THE DESCRIPTION

Rewrite the two paragraphs starting on page 15, line 20, as follows:

Bayesian networks are directed acyclical graphs (DAG) in which the nodes correspond to (stochastic) variables. The arcs describe a direct causal relationship between the linked variables. The strength of these links is given by conditional probability distributions (cpds). More formally, let the set  $A = \{x_1, \dots, x_N\}$  of  $N$  variables define a DAG. For each variable  $x_i$ ,  $i=1, \dots, N$ , there exists a sub-set of variables of  $\Omega$ ,  $\theta_{-i} = \Pi_{x_i}$ , the parents set of  $x_i$  i.e., the predecessors of  $x_i$  in the DAG, such that  $P(x_i | \theta_{-i}) = P(x_i | x_1, \dots, x_{i-1})$ , where  $P(\cdot | \cdot)$  is a cpd, strictly positive. Now, given the joint probability density function (pdf)  $P(x_1, \dots, x_N)$ , using the chain rule:  $P(x_1, \dots, x_N) = P(x_N | x_{N-1}, \dots, x_1) \times \dots \times P(x_2 | x_1) \times P(x_1)$ . According to this equation, the parent set  $\theta_{-i} = \Pi_{x_i}$  has the property that  $x_i$  and  $\{x_1, \dots, x_N\} \setminus \Pi_{x_i}$  are conditionally independent given  $\theta_{-i} = \Pi_{x_i}$ .

In FIG. 4, the flow diagram of the BE has the structure of a DAG made up of three layers. In each layer, each element corresponds to a node in the DAG. The directed arcs join one node in a given layer with one or more nodes of the preceding layer. Two sets of arcs join the elements of the three layers. For a given layer and for a given element we compute a joint pdf as previously described. More precisely, for an element (node)  $i^{(l)}$  associated with the  $l$ -th layer, the joint pdf is:

$$\begin{aligned}
 & P^{(2)}(\underline{x}_{i(1)}^{(1)}, \underline{g}_{i(1)}^{(1)}, \dots, \underline{g}_{i(1)}^{(2)}) = P(\underline{x}_{i(1)}^{(1)} | \underline{g}_{i(1)}^{(1)}) \\
 * & \{P(\underline{x}_{i(1)}^{(1)} | \underline{g}_{i(1)}^{(1)}) \dots P(\underline{x}_{N(1-1)}^{(1)} | \underline{g}_{N(1-1)}^{(1)})\} \dots \\
 * & \{P(\underline{x}_{i(2)}^{(2)} | \underline{g}_{i(2)}^{(2)}) \dots P(\underline{x}_{N(2)}^{(2)} | \underline{g}_{N(2)}^{(2)})\}, \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 & P^{(1)}(\underline{x}_{i(1)}^{(1)}, \Pi^{(1-1)}, \dots, \Pi^{(2)}) = P(\underline{x}_{i(1)}^{(1)} | \Pi^{(1)}) \\
 \times & \{P(\underline{x}_{i(1)}^{(1)} | \Pi^{(1-1)}) \dots P(\underline{x}_{N(1-1)}^{(1)} | \Pi^{(1-1)})\} \dots \\
 \times & \{P(\underline{x}_{i(2)}^{(2)} | \Pi^{(2)}) \dots P(\underline{x}_{N(2)}^{(2)} | \Pi^{(2)})\}, \quad (1)
 \end{aligned}$$

where for each element  $\underline{x}_{i(1)}^{(1)}$  there exists a parent set  $\Pi_{i(1)}^{(1)}$ , the union of the parent sets for a given level  $l$ , i.e.,  $\underline{g}_{i(1)}^{(1)} = \bigcup_{j=1}^{N(1)} \underline{g}_{j(1)}^{(1)}$ ;  $\Pi^{(1)} \equiv \sum_{i=1}^{N(1)} \Pi_{i(1)}^{(1)}$ . There can exist an overlap between the different parent sets for each level.